Observing the expanding universe

Cosmology Block Course 2013

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Contents

What can we observe?

In the previous lectures, we learned about the **Friedmann-Lemaître-Robertson-Walker** (FLRW) models of the expanding universe.

Those models have free parameters: $H_0, \Omega_M, \Omega_R, \Omega_\Lambda$.

The parameters need to be fixed $\Rightarrow$ this specifies our world model

Also, the resulting model needs to be tested.

Two fundamental ways of measuring distances

- Deduce distance from known length scale (e.g. parallax)
- Deduce distance from known luminosity (standard candle methods)

Both involve the geometry of space. Are they influenced by universal expansion, as well?

Comoving and proper distance

Recall the FRW metric:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2\left[\frac{\mathrm{d}r^2}{1-Kr^2} + r^2\mathrm{d}\Omega\right] = -\mathrm{d}t^2 + a(t)^2\tilde{g}(\vec{x})_{ij}\mathrm{d}x^i\,\mathrm{d}x^j$$

In this coordinate system, galaxy locations *up to scale* can be described by radial coordinate values: **comoving distance**. Good to keep track of where galaxies go!

"Instantaneous distances": stop the universe and measure with a ruler. These are the distances at a fixed time as described by the spatial part of the metric: **proper (spatial) distance**

Co-moving distances related to redshift

From FRW metric and $\mathrm{d}s^2 = 0$, for light propagation

$$\int \frac{\mathrm{d}t}{a(t)} = \pm \int \frac{\mathrm{d}r}{\sqrt{1 - Kr^2}} = \pm \begin{cases} \arcsin(r) & K = +1 \\ r & K = 0 \\ \operatorname{arsinh}(r) & K = -1 \end{cases} .$$

Consider a source at radial coordinate $r(z)$ whose light reaches us with redshift z:

$$\int_{t(z)}^{t_0} \frac{\mathrm{d}t}{a(t)} = \frac{1}{a_0 H_0} \int_{1/(1+z)}^{1} \frac{\mathrm{d}x}{x^2 \sqrt{\Omega_\Lambda + \Omega_K x^{-2} + \Omega_M x^{-3} + \Omega_R x^{-4}}}.$$

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Co-moving distances related to redshift

$$
\begin{aligned}
r(z) &= S\left[\int_{t(z)}^{t_0} \frac{\mathrm{d}t}{a(t)}\right] \\
&= S\left[\frac{1}{a_0 H_0} \int_{1/(1+z)}^{1} \frac{\mathrm{d}x}{x^2\,\sqrt{\Omega_\Lambda + \Omega_K\,x^{-2} + \Omega_M\,x^{-3} + \Omega_R\,x^{-4}}}\right]
\end{aligned}
$$

where

$$
S[y] \equiv \begin{cases} \sin y & K = +1 \\ y & K = 0 \\ \sinh y & K = -1 \end{cases}
$$

Proper distance related to redshift

Use

$$\Omega_K = -\frac{K}{a_0^2 H_0^2}$$

and $\sinh ix = i \sin x$ to re-write as

$$d_{\rm now}(z) = a_0 r(z)$$

$$= \frac{1}{H_0\sqrt{\Omega_K}} \cdot \sinh\left[\sqrt{\Omega_K} \int\limits_{1/(1+z)}^{1} \frac{\mathrm{d}x}{x^2\sqrt{\Omega_\Lambda + \Omega_K x^{-2} + \Omega_M x^{-3} + \Omega_R x^{-4}}}\right]$$

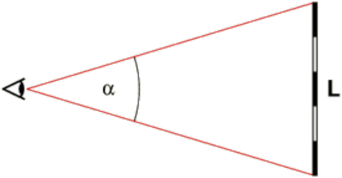
Light travel time

Determine travel time by using earlier expression relating $\mathrm{d}t$ and $\mathrm{d}x$ and integrating up:

$$t_0 - t(z) = \frac{1}{H_0} \int_{1/(1+z)}^{1} \frac{\mathrm{d}x}{x\,\sqrt{\Omega_\Lambda + \Omega_K\,x^{-2} + \Omega_M\,x^{-3} + \Omega_R\,x^{-4}}}.$$

Angular distance

Consider an object at redshift z with (proper) size L:



Under what angle will we see that object? Go back to FRW metric:

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta \, d\phi^2) \right].$$

We've seen how light with ds^2 travels in the radial direction. Consider two *light rays* reaching us with a (small) angular difference α.

Angular distance

Now consider the time t_1 when the light was emitted. Use the metric and insert the angular difference $\Delta\theta$:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \left[\frac{\mathrm{d}r^2}{1 - Kr^2} + r^2 (d\theta^2 + \sin^2\theta \, \mathrm{d}\phi^2) \right].$$

$\Rightarrow \mathrm{d}s = a(t_1) r_E \, \alpha = L$.

Define **angular distance** analogously to classical geometry:

$$d_A(z) = \frac{L}{\alpha} = a(t_1)\, r_E(z) = \frac{a_0}{1+z}\, r_E(z) = \frac{d_{\mathrm{now}}}{1+z}$$

(cf. explicit formula for d_{now} calculated earlier).

Classical luminosity distance

Absolute luminosity L is total energy emitted by an object per second.

Apparent luminosity (energy flux) f is the energy received per second per unit area.

For isotropic brightness: total energy passes through sphere with radius r, so

$$f = \frac{L}{4\pi r^2}.$$

If L is the same for each object in a certain class, or can be determined from observations, we have a **standard candle**.

FRW luminosity distance

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \right].$$

Corrections to classical derivation for light emitted at time t_1 by object at redshift z:

- Energy emitted at time t_1 has spread out on sphere with proper area $4\pi r_1(z)^2 a^2(t_0)$ (use symmetry between the object's and our own position)
- Photons arrive at a lower rate, given by redshift factor $a(t_1)/a_0 = 1/(1+z)$
- Photon energy is $E = h\nu$; redshift reduces energy by $1/(1+z)$

FRW luminosity distance

Result:

$$f = \frac{L}{4\pi r_1(z)^2 a_0^2 (1+z)^2}$$

Define luminosity distance by

$$f = \frac{L}{4\pi d_L(z)^2},$$

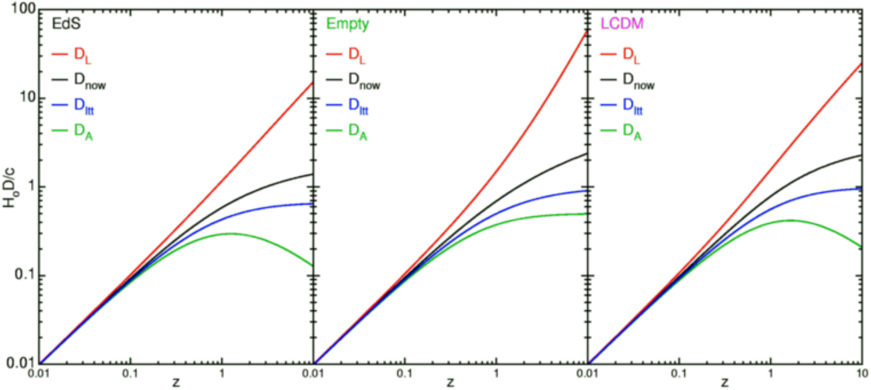
so

$$d_L(z) = a_0 r_1(z) \cdot (1+z) = d_A(z) \cdot (1+z)^2.$$

Different notions of distance

1. **redshift** z — for monotonously expanding universe, good distance measure; model-independent, can be measured directly
2. **proper distance** d_{now} — instantanous distance
3. **co-moving distance** r — coordinate distance, useful for tagging
4. **light-travel time** — the original light year
5. **angular distance** — ties in with observation of standard rulers
6. **luminosity distance** — ties in with observations of standard candles

Different notions of distance



From: Ned Wright's cosmology tutorial

The central problem of astronomy

The central problem of astronomy: the third dimension!

Angular distances are fairly easy to measure precisely — up until the late 19th century, astronomy was *positional astronomy*.

Lunar and planetary parallaxes: Cassini & Richer, 1672, Mars

Stellar parallax: Bessel 1838, 61Cyg, 11.4 ly

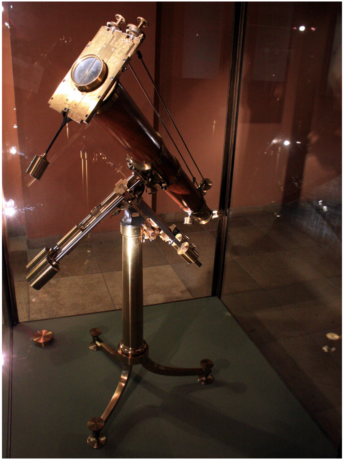
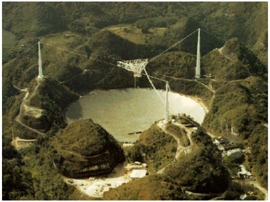


Image:
Small heliometer
Utzschneider & Fraunhofer 1820,
Deutsches Museum München

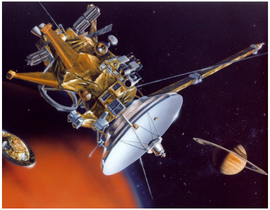
Distances within our Solar System

Within our Solar System: length scale is the *astronomical unit* (Kepler scaling!)

Modern determination directly with *radar distances* to planets and *telemetry* from planetary probes in comparison with ephemeris data.



Arecibo observatory



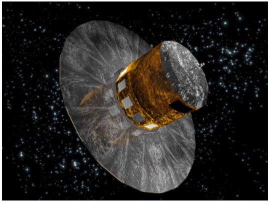
Cassini mission

Stellar parallaxes

First step: **parallax measurements**

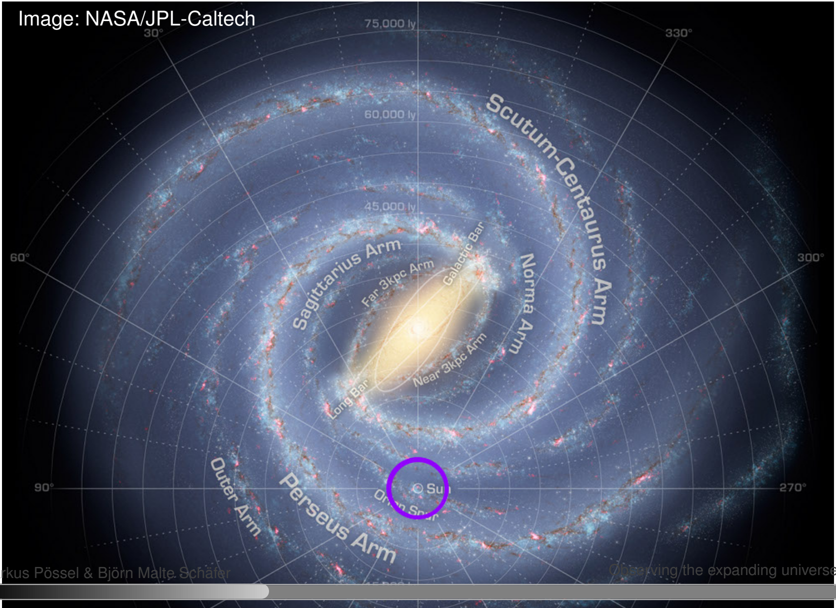
Dominated by satellite missions: Hipparcos 1989-1993, Gaia slated for launch October 2013

(image on the right; credit: ESA)



Quantity	Hipparcos	Gaia
Accuracy	1 mas	20 μas (at 15 mag)
Distances to 10%	100 pc	5 kpc

How far will stellar parallaxes get us?



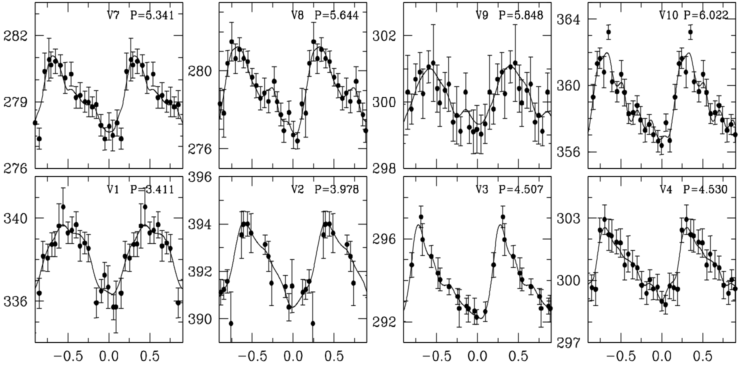
Other parallax measurements

- **Kinematical parallax**: Co-moving (e.g. open cluster) in same direction. With proper motion and Doppler motion, reconstruct distance.
- **Statistical parallax**: Group of stars with known relative distances (e.g. at same distance). Assume that Doppler shifts and proper motion are connected
- Cepheids pulsating: compare change in angular size (interferometry) with change in radial velocity (Doppler), out to 400 pc or so (Lane et al. 2000, Kervella et al. 2004)
- Tracking orbits around central mass with proper motion and Doppler shift — infer scale. Example: stars around central black hole of the Milky Way

Luminosity measurements: Cepheids

Henrietta Leavitt

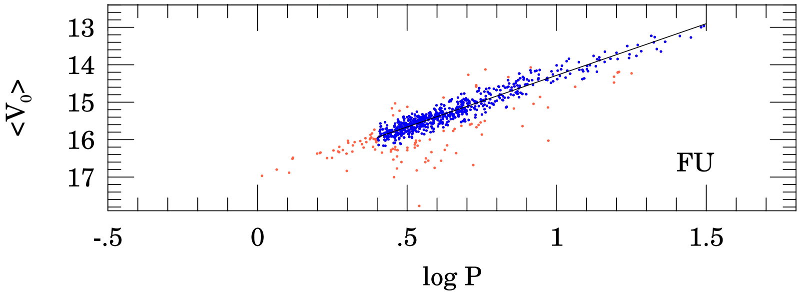
Cepheid light curves



Cepheid light curves — measurements show: pulsating stars (even possible to distinguish fundamental mode and overtones!)

Figure from Joshi et al. 2010

Cepheid period-luminosity relation



Period-luminosity relation $m = -2{,}76 \cdot \log(P/d) + 17{,}042$ for Great Magellanic Cloud

Part of Fig. 3 in Udalski 1999 in *Acta Astronomica* **49**, 201

Other luminosity distances

- **RR Lyrae**: Shorter-period variable stars (1/10 to 1 day) with period-luminosity-relation
- **Main sequence**: Shape and scale given by physical quantities. Calibrate with parallax measurements. Look at distant star clusters. (Related: red clump stars in color-magnitude diagrams.)
- **Eclipsing binary** with smaller companion: Doppler shift gives velocity; time for companion to pass primary star gives primary star diameter; spectroscopy gives temperature; area and Stefan-Boltzmann law give absolute luminosity

Secondary distance indicators

Relations that have been calibrated using primary indicators. Typically on the scale of galaxies.

- **Tully-Fisher relation**: (empirical) relation for spiral galaxies: widening of 21 cm line → maximum speed of rotation → correlated with mass of galaxy → correlated with absolute luminosity
- **Faber-Jackson relation**: dispersion of stellar velocities → galaxy mass (virial theorem) → absolute luminosity
- **Fundamental plane**: add surface brightness to the correlation; in this three-dimensional space, galaxies are distributed along a two-dimensional plane (Faber-Jackson is then a projection)
- **Surface brightness fluctuations**: For more distant galaxies, the Poisson fluctuation due to surface brightness being made up of individual stars is less (smeared out)

. . . and the arguably most important one: Sn Ia!

Supernovae of Type Ia

Standard model (there might be several): Accretion of matter onto a White Dwarf.

Stability limit: *Chandrasekhar mass* at 1.44 $M_\odot$ – once that is reached, thermonuclear explosion.

Characteristic light curve – dominated by radioactive decay of Ni-56 to Co-56 to Fe-56.

Light curves of Supernovae of Type Ia

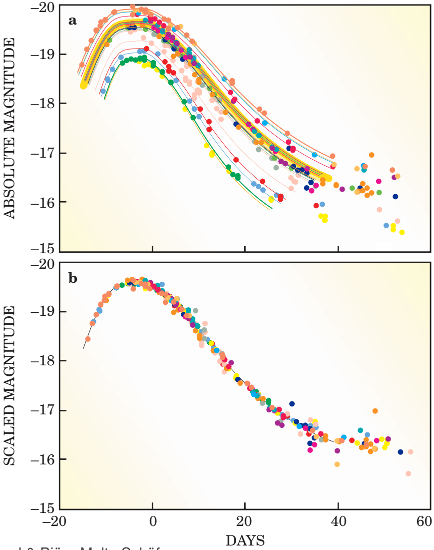


Image: Perlmutter 2003 in *Physics Today*

Tracing cosmic expansion

The old way: Recall expansion for *lookback time* $t_0 - t_1$ (time a signal has travelled):

$$z = H_0(t_0 - t_1) + \frac{1}{2}(q_0 + 2)H_0^2(t_0 - t_1)^2 + O((t_0 - t_1)^3)$$

with $H_0 = \dot{a}(t_0)/a_0$ and $q_0 = -\ddot{a}(t_0)/(H_0^2 a_0)$.

Invert to obtain

$$H_0(t_0 - t_1) = z - \frac{1}{2}(q_0 + 2)z^2 + O(z^3).$$

Hubble relation for the luminosity distance

Taylor-expand

$$\int_{t_1}^{t_0} \frac{\mathrm{d}t}{a(t)} = \int_0^{r_1} \frac{\mathrm{d}r}{\sqrt{1-Kr^2}}$$

as

$$r_1 = \frac{t_0 - t_1}{a_0} + \frac{1}{2}\frac{H_0}{a_0}(t_0 - t_1)^2 + \ldots$$

to obtain proper distance

$$r_1 a_0 = \frac{1}{H_0}\left[z - \frac{1}{2}(1+q_0)z^2 + \cdots\right]$$

and from that luminosity distance

$$d_L(z) = \frac{1}{H_0}\left[z + \frac{1}{2}(1-q_0)z^2 + \ldots\right]$$

What can we measure?

- H_0 sets basic cosmic time scale
- q_0 gives us

$$q_0 = \frac{1}{2}(\Omega_M - 2\Omega_\Lambda + 2\Omega_R)$$

- Any curvature measurement gives us

$$\Omega_\Lambda + \Omega_M + \Omega_R + \Omega_K = 1$$

(this will become later on with the cosmic background radiation)

Luminosity distance-redshift relation

The modern view: Model directly with basic parameters!

$$d_L(z) = a_0(1+z)\cdot S\left[\frac{1}{a_0 H_0}\int_{1/(1+z)}^{1}\frac{\mathrm{d}x}{x^2\,\sqrt{\Omega_\Lambda + \Omega_K\,x^{-2} + \Omega_M\,x^{-3} + \Omega_R\,x^{-4}}}\right]$$

where

$$S[y] \equiv \begin{cases} \sin y & K = +1 \\ y & K = 0 \\ \sinh y & K = -1 \end{cases}$$

Hubble's original measurements

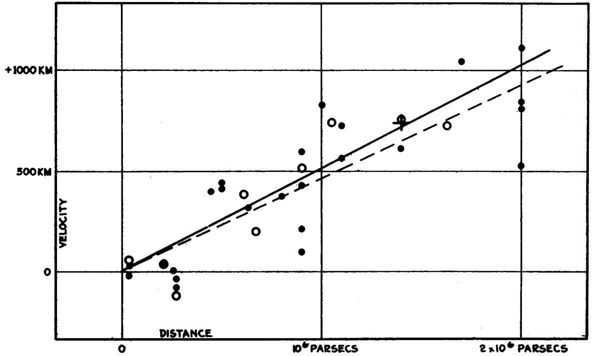
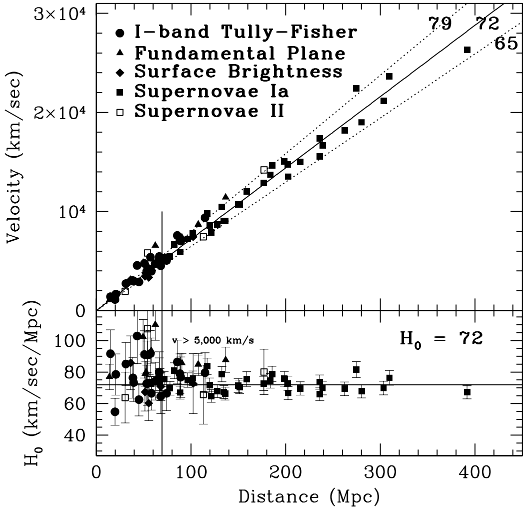


FIGURE 1
Velocity-Distance Relation among Extra-Galactic Nebulae.

Hubble 1929: "A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae" in PNAS 15(3), S. 168ff.

HST Key Project results



From Freedman 2001 et al. (HST Key Project)

Putting it all (almost) together

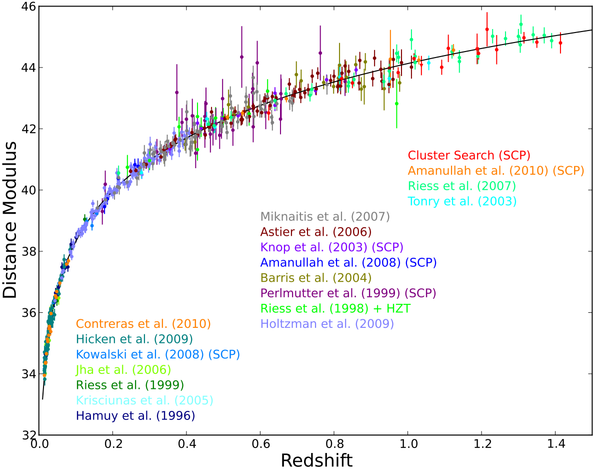
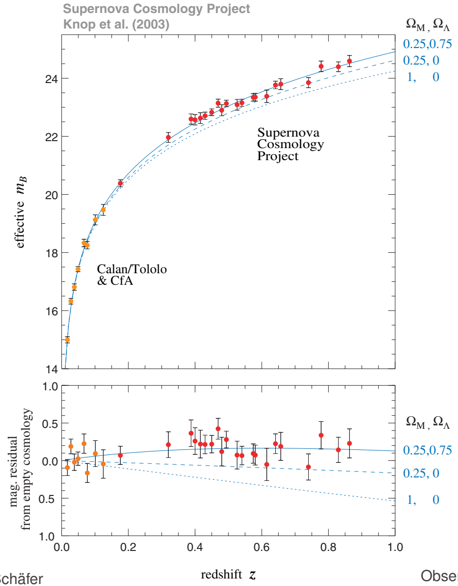


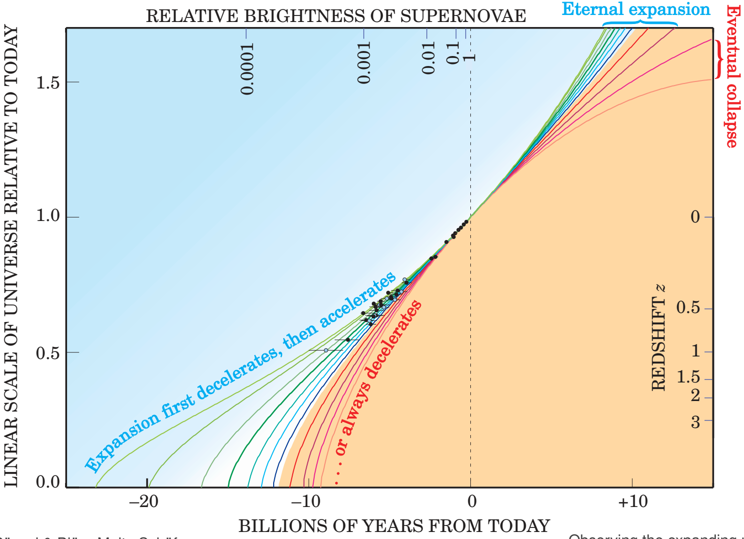
Image: Suzuki et al. 2011

The high-z regime



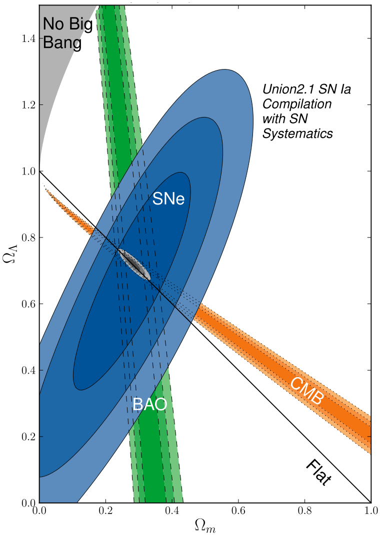
Reconstructing cosmic history

Perlmutter, *Physics Today* 2003



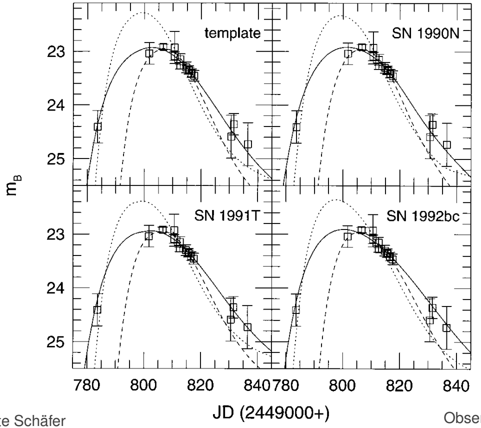
Supernova Cosmology Project Plot

Suzuki et al. 2011



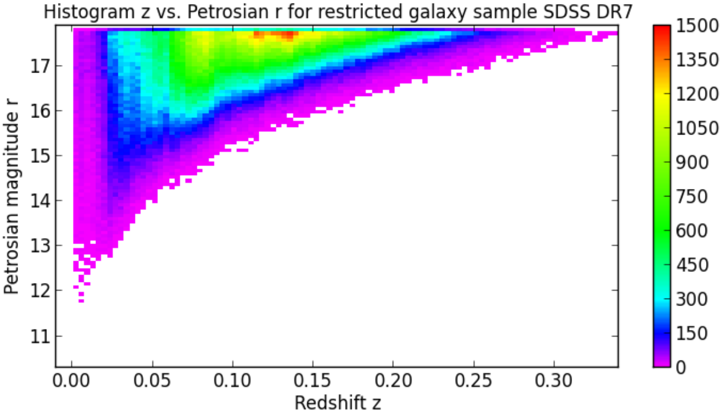
How to tell expansion from tired light?

- Test $d_L(z)/d_A(z) = (1+z)^2$: Tolman's surface brightness test (Lubin and Sandage 2001; complicated by galaxy evolution)
- Time dilation in supernova light-curves (Leibundgut et al. 1996):



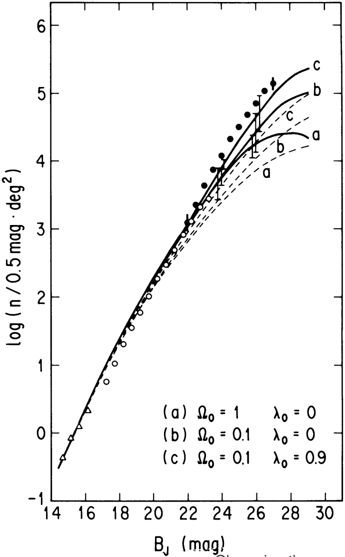
Tracing expansion by numbers

Apparent magnitudes for galaxies (careful with evolution effects):



Tracing expansion by numbers

Fukugita et al. 1990: Number counts of faint galaxies; simple evolution model with parameters included. Hints of $0.5 > \Omega_\Lambda > 1$.



Age determinations

Trivially, nothing in the universe can be older than the universe itself.

(There was a time when that appeared to be a problem!)

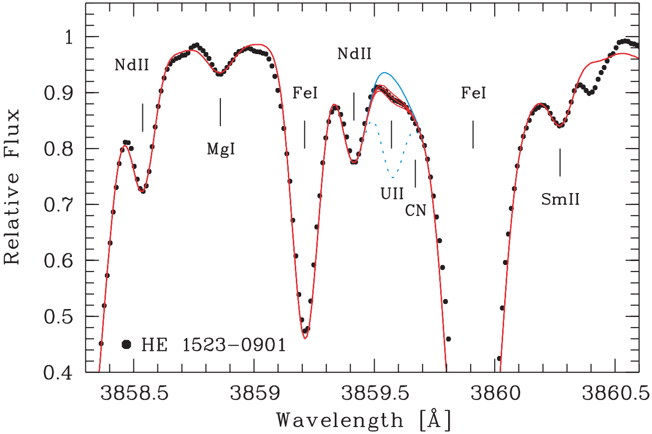
First possibility: Radioactive dating. Some half-life values:

^{235}U	$7 \cdot 10^8$ a
^{232}Th	$1.4 \cdot 10^{10}$ a

$\Rightarrow$ Heavy elements formed in the r-process (rapid addition of neutrons) in core-collapse supernovae (some modelling involved!)

HE 1523-0903

Example for very old, metal-poor star (Frebel, Christlieb et al. 2007): U- and Th- dated to 13.2 Gyr!

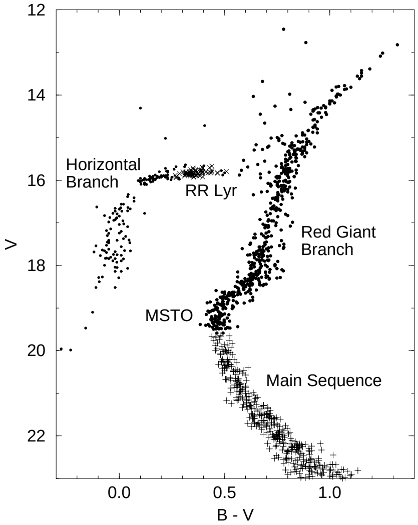


Stellar ages

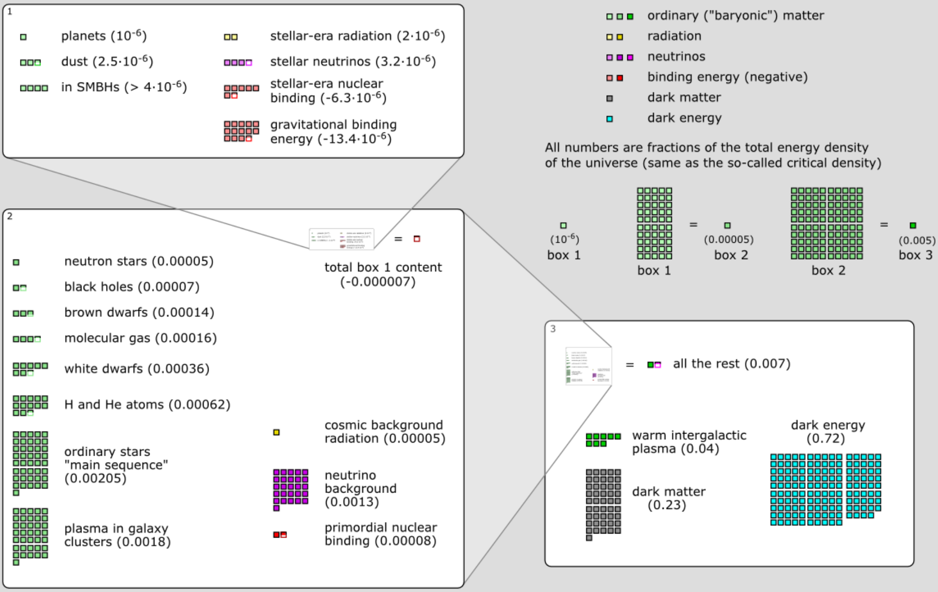
Model for stellar evolution: stars move in the *Hertzsprung-Russell diagram* (color-magnitude diagram) as they evolve.

Lifetime $\tau \sim L^{-2/3}$, $L \sim M^3$ and $\tau \sim T^{-1}$.

Oldest globular clusters give 13.2 ± 2 Gyr (Carretta et al. 2000).



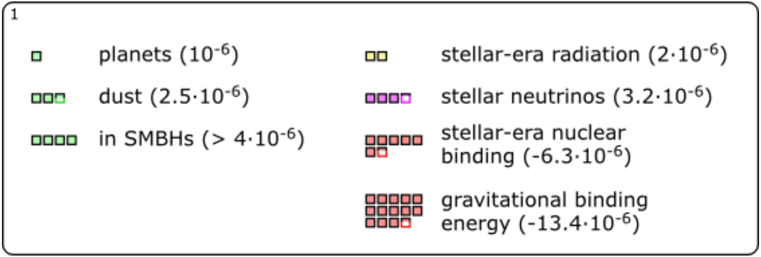
The Cosmic Energy Inventory (current era)



Markus Pössel & Björn Malte Schäfer

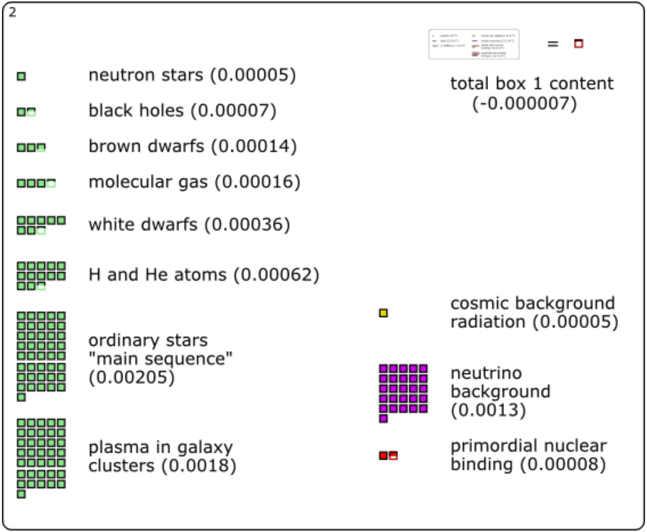
Chart style following Randall Munroe's xkcd.com/radiation

Cosmic inventory: Small scales

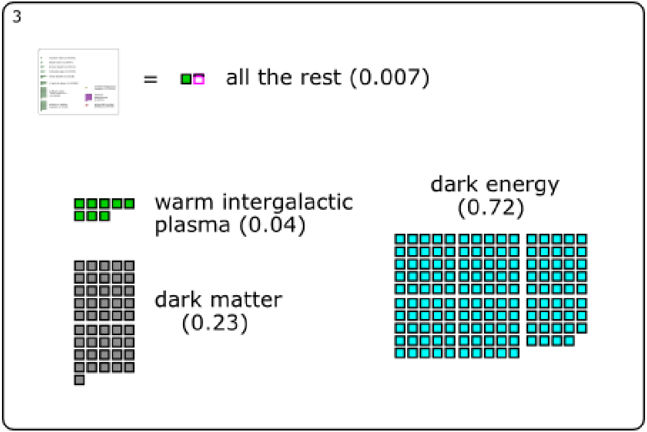


All numbers are fractions of the critical density ρ_{c0}. Numbers from Fukugita & Peebles 2004.

Cosmic inventory: Medium scales



Cosmic inventory: Large scales



Estimating Ω_M

- Virial theorem to measure galaxy cluster mass; derive mass-to-light ratio; from total luminosity: $\Omega_M \approx 0.3$ (e.g. Yasuda et al. 2004)
- Warm plasma: difficult to detect (not accessible via X-rays); mainly used to balance the budget
- Later on, CMB and weak lensing will also have something to say

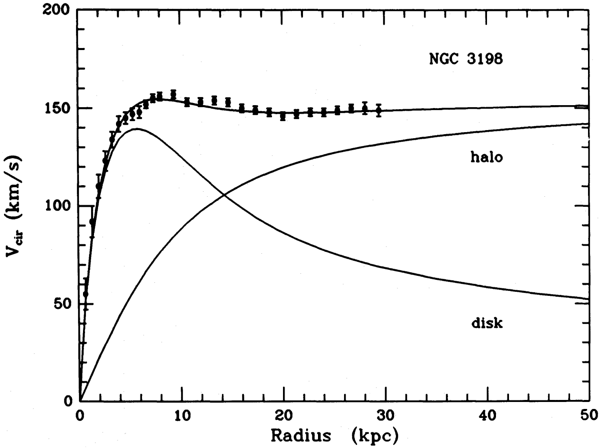
Intriguingly, much of Ω_M seems to be in some form other than ordinary (baryonic) matter!

Dark matter

- no electromagnetic interaction, just gravitational
- first postulated by Fritz Zwicky to explain motion within galaxy clusters (virial theorem)
- direct detection experiments: inconclusive, but promising
- WIMPs: particles based on supersymmetric extensions? $\Rightarrow$ *LHC*
- so far, we did not differentiate Ω_M into dark and luminous matter, but this will become important in the early universe

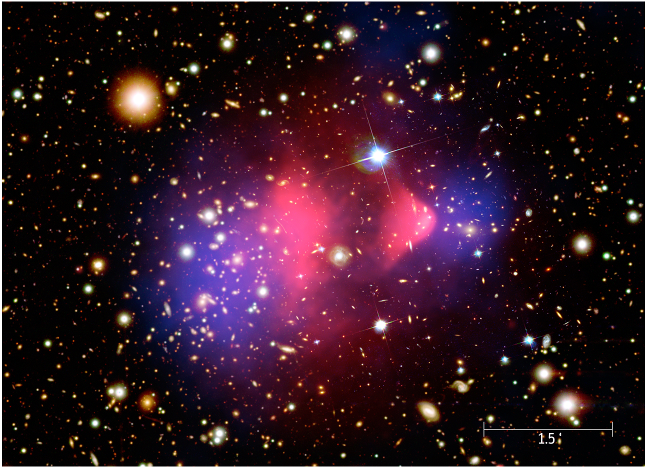
Dark matter: Rotation curves

Deviation from Kepler potential as generated by visible contributions to mass (here van Albada et al. 1985):



Dark matter: Lensing & Collisions

Bullet Cluster (NASA/CXC/M. Weiss): Tracing dark matter with gravitational lensing



Next stop: the early universe

. . . with sundry additional possibilities for parameter determination and consistency checks.

Literature

d'Inverno, Ray: *Introducing Einstein's Relativity*. Oxford University Press 1992.

Weinberg, Steven: *Cosmology*. Oxford University Press 2008 [main source for this lecture!]

Wright, Ned: *Cosmology tutorial* at http://www.astro.ucla.edu/ wright/cosmolog.htm