Exercises, Day 3

Markus Pössel (HdA) and Björn Malte Schäfer (ITA), 31 July 2013

Names: ____

1 Properties of Friedmann-Robertson-Walker spaces

(a) Show that the worldline of a particle at rest in the standard cosmological reference frame in a space with Friedmann-Robertson-Walker (FRW) metric is a geodesic.

(b) Derive the equation for $\dot{\rho}$ for an ideal fluid at rest in the cosmic reference frame in FRW-spacetime from energy conservation, $(\nabla_\mu T)^{\mu 0} = 0$.

2 Newtonian dynamics of an expanding universe

Assume that Euclidean space ($K = 0$) is filled with matter whose mass density ρ is the same everywhere. Assuming that matter in this space is undergoing expansion governed by a scale factor $a(t)$ (that is, all distances change proportionally to $a(t)$ over time), derive a differential equation for $a(t)$ using Newton's laws of mechanics and of gravity. If possible, re-write as a first-order differential equation.

To do so, pick two locations in space; choose one as the origin of your coordinate system. You could imagine that those are the locations of two of your galaxies. Assume that the galaxy at the origin is at rest, and that the other is moving in a manner prescribed by the expansion. Which force(s) act(s) on the moving galaxy? For the calculation, divide space into two regions: the sphere around the origin on whose surface the moving galaxy is located, and the outside of that sphere. Does gravitational attraction from the matter outside contribute anything to the force? What does the gravitational influence on the moving galaxy of all the matter inside the sphere add up to? (To answer those two last questions, you need not explicitly calculate the simplifications that arise; you should argue from your understanding of Newtonian gravity.)

3 Dynamics of our own universe

For the simplified Friedmann equation with parameters C, Λ, K, consider the case that includes our own universe, namely $\Lambda > 0$ and $K = 0$. Find an explicit solution. This will involve substitutions

$$u(t) = \frac{2\Lambda}{3C}\, a(t)^2 \tag{1}$$

and, later on, $v(t) = u(t) + 1$ and $\cosh[w(t)] = v(t)$. Assume that you already know that this model involves a big bang singularity $a(t_i) = 0$, and choose your time coordinate so that $a(t = 0) = 0$, as this will make the integration somewhat simpler.

How is the Hubble constant H_0 related to the parameters C, Λ in this case?